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A LAMINAR FLOW ELEMENT WITH A LINEAR PRESSURE DROP VERSUS VOLUMETRIC FLOW

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ABSTRACT

The pressure drop across a typical laminar flow element consists of a quadratic entrance function, a linear function due to the fully developed flow and a quadratic exit function. These nonlinear terms are also functions of fluid density. If a strictly linear function is assumed then the nonlinear terms introduce an error into the flow measurement that changes with the fluid temperature, pressure and composition. This limits the accuracy of the flow reading.

An annular flow passage alone or in parallel with other laminar flow passages can remove these unstable, nonlinear errors. A pressure sense point can be inserted into an annular passageway sufficiently downstream of the inlet point such that the flow is a fully developed laminar flow at this position. Another pressure sense point can be inserted downstream of the first sense point, but sufficiently far upstream of the exit point such that there will be no disturbance in the flow from the exit point. There is only fully developed laminar flow between these two points.

NOMENCLATURE

A cross-sectional area of passage
c_f friction coefficient
D diameter of passage
D_h hydraulic
L length
L_e = entrance length
K_c = compression coefficient
K_e = expansion coefficient
P = pressure
Q = volumetric flow rate
r = radius
Re = Reynold's number
V = gas velocity
ΔP = differential pressure drop
ΔP_a = differential pressure drop due to area change
Δr = distance from inner diameter to outer diameter of annular passage

greek letters
τ_o = shear stress on surface
μ = gas viscosity
ρ = gas density

Subscripts
i inlet condition
o outlet condition

INTRODUCTION

A laminar flow element is a device that will generate a differential pressure that is a mostly linear function of the volumetric flow passing through it. The laminar flow element does this by dividing the flow up into many parallel flow passages that have very small diameters. Since only a small flow passes through each passage, the Reynolds number of the flow through each passage is low enough that laminar flow is maintained throughout the measurement range.

These devices can be used as a volumetric flow measurement device. This can be achieved by measuring the differential pressure across the laminar flow element generated by the flow passing through it and correcting for changes in the gas viscosity caused by temperature. This corrected differential pressure is then used as the independent variable in a polynomial fit to determine the volume of the flow passing through the midpoint of the laminar flow element.

Another common use for these devices is as a flow shunt inside a thermal mass flow controller. The thermal mass flow sensor has a typical full scale range that is less than 10 standard cm3/min. A flow shunting arrangement is used to overcome this limitation. A flow shunt is placed in parallel with the sensor. The differential pressure created across the flow shunt provides the driving force to send a small portion of the total flow through the sensor. The sensor measures the mass flow rate of this diverted flow stream and provides an output proportional to measured mass flow.

If the mass flow rate of the main flow stream is measured externally during calibration, a multiplier can be calculated from the ratio of the sensed flow and the total flow. This multiplier can then be used to calculate the total flow from the measured flow during normal operation. Ideally this flow division between the diverted flow stream and the main flow is constant. This would occur if the pressure drop versus volumetric flow rate curve of the diverted flow path has the same shape as the pressure drop versus volumetric flow rate of the main flow path; i. e. they differ only by a constant multiplier of the volumetric flow rate that is independent of temperature, pressure, or gas variations or flow rates.

In practice, most of the flow shunting arrangements do not have the same pressure drop versus volumetric flow curve shape as the sensors used across them. The sensor is typically constructed from a thin capillary tube which is 100 times longer than its diameter. This tube has a pressure drop versus volumetric flow curve that is mostly linear with a very small quadratic term from the entrance and exit effects. The shunt however, is often shorter than the sensor with larger flow paths. Often the flow paths contain sharp bends and varying flow areas. The pressure drop versus volumetric flow drop curve for these shunts is often dominated by second order terms that are functions of the gas variables. This makes the flow multiplier a function of the gas density, viscosity and flow rate. This is typically overcome by measuring the flow multiplier at several different flow rates with a gas that has a viscosity that is similar to the gas that will be measured in normal operation. A linear extrapolation is used to calculate the flow multiplier at all points between the measured flow points.

THEORY

A flow divider for a thermal mass flow transducer usually consists of an inlet plenum, a flow restriction and an outlet plenum. (See Figure 1.) Since stability of the flow multiplier is desired to ensure a stable instrument, there must be some matching between the linear volumetric flow versus pressure drop of the sensor and the shape of the volumetric flow versus pressure drop of the flow restriction. Most instruments employ Poiseuille's law[1] and use some sort of multi-passage device that creates laminar flow between the upstream sensor inlet and the downstream outlet. This makes the volumetric flow versus pressure drop curve primarily linear, but there are other effects which introduce higher order terms.

Most flow transducers are designed such that the outlet plenum has a smaller diameter than the inlet plenum. This eases the insertion and containment of the flow restriction between the sensor inlet point and the sensor outlet point. If we were to remove the flow restriction we know the energy of the gas must be conserved when passing from the inlet plenum to the outlet plenum. From Bernoulli's[2] equation we know that sum of the kinetic energy and the pressure at each point must be a constant and since all of the pressure drops are small we can assume the flow is incompressible.

$$P_i + \frac{1}{2}\rho V_i^2 = P_o + \frac{1}{2}\rho V_o^2 \qquad (1)$$

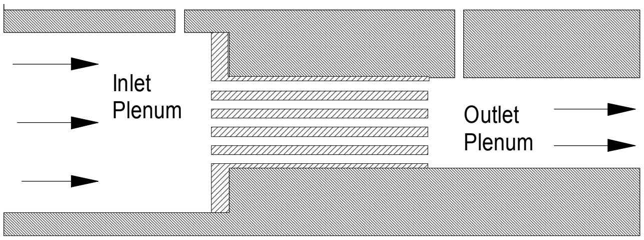

Figure 1 Typical Flow Divider

From conservation of mass we know that:

$$\rho V_i A_i = \rho V_o A_o \qquad (2)$$

$$V_o = \frac{A_i}{A_o} V_i \qquad (3)$$

Combining these two equations we get:

$$P_i - P_o = \frac{1}{2}\rho V_i^2 \left[\left(\frac{A_i}{A_o} \right)^2 - 1 \right] \qquad (4)$$

Substituting in the function of the diameters for the area of the flow passage and simplifying we have:

$$\Delta P_a = \frac{1}{2}\rho V_i^2 \left[\left(\frac{D_i}{D_o} \right)^4 - 1 \right] \qquad (5)$$

We can see that even with no effect from the flow restriction there will be a pressure drop between the sensor inlet and outlet points. This pressure drop will be a strong function of the ratio of the two diameters. Since the drop is a square function of the flow velocity the differential pressure will be non-linear with respect to flow rate. Note also that the pressure drop is a function of density. The density will vary as

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[1] Bermand, Armand, 1990, "Vacuum engineering calculations, formulas, and solved exercises", Academic Press, San Diego

a function of system pressure and it will also vary when the gas composition changes. This will cause the magnitude of the pressure drop due to the area change to be a function of system pressure and gas composition.

Most of the flow restrictions used contain or can be approximated by many short capillary tubes in parallel. From Rimberg[1] we know that the equation for the pressure drop across a capillary tube contains terms that are proportional to the square of the volumetric flow rate. These terms come from the pressure drops associated with the sudden compression at the entrance and the sudden expansion at the exit of the capillary tube. The end effect terms are a function of density which will cause the quadratic term to vary with system pressure and gas composition. The absence of viscosity in the second term will cause a change in the relative magnitudes of the two terms whenever the viscosity of the flowing gas changes.

$$\Delta P = \frac{128\mu L Q}{\pi D^4} + \frac{8\rho Q^2}{\pi^2 D^4}\left(K_c + K_e\right) \tag{1}$$

The end effect for a typical laminar flow element in air account for approximately 4% of the total pressure drop. For hydrogen, however, which has a density that is about 14 times less than air and has a viscosity that is much greater than air, the second term is completely negligible. For the heavier gasses such as sulfur hexaflouride which has a density 5 times that of air the end effects will become 10% of the total. These changes make it impossible to accurately calibrate an instrument on one gas and use it for another gas.

The pressure drop is linear with respect to the volumetric flow rate between a point that is downstream of the entrance area and another point further downstream but upstream of the exit region. From Kays & Crawford[2] we know that entrance length of a capillary tube in laminar flow is a function of the Reynolds number and the tube diameter.

$$L_e = D\frac{\mathrm{Re}}{20} \tag{2}$$

Where the Reynolds number for a tube is:

$$\mathrm{Re} = \frac{DV\rho}{\mu} \tag{3}$$

and

$$V = \frac{Q}{A} = \frac{4Q}{\pi D^2} \tag{4}$$

Therefore:

$$L_e = \frac{Q\rho}{5\pi\mu} \tag{5}$$

For a typical flow divider tube the entry length is approximately 0.16 cm. From this it can be seen that if the sensor inlet pickup point could be inside of the flow divider tube but downstream of the entrance length and if the sensor outlet point were also inside the flow divider tube but upstream of the exit point then the pressure drop that drives the flow through the sensor would be linear with respect to volumetric flow rate. Since the pressure drop across the sensor now increases linearly with the main flow rate and the sensor has a linearly increasing flow with respect to pressure drop, there is now a flow through the sensor which is directly proportional to the main flow through the flow divider, without the flow division errors that are present when the sensor samples the flow completely upstream and downstream of the flow divider.

Unfortunately, a typical flow divider tube has an internal diameter on the order of 0.3 mm. This is too small to insert tap points into the tube. Also, the sample flow through the sensor is approximately 10 cm^3/min while the flow through a divider tube is approximately 25 cm^3/min. This means the sample flow would be affecting the flow it was trying to measure. If the sensor tube is made large enough, and with enough flow through it to insert the sensor taps at these positions, then the pressure drop would be too small to push the necessary flow through the sensor tube.

The solution is to use a different geometry for the flow tube. It must be large enough to allow the sample points in the middle yet with passages thin enough to create the differential pressures required for the sensor. An annular passage meets these requirements.

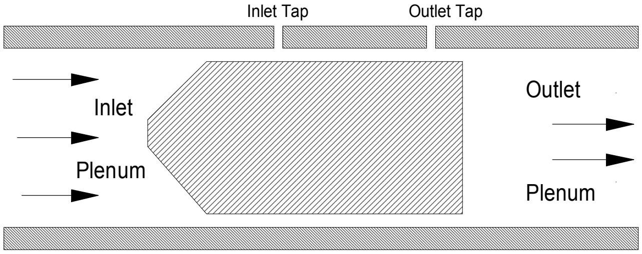

Figure 1 Cross sectional view of annular flow divider

The basic operation is similar to the operation of the tubular flow divider but the equations for the entry length and pressure drop will be different. If we assume that the annular

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[1] Rimberg, D. "Pressure Drop Across Sharp-end Capillary Tubes," I&EC Fundamentals, vol.6, no. 4, November 1967

[2] Kays, W. M., Crawford M. E., 1993, "Convective Heat and Mass Transfer", 3rd edition, McGraw-Hill, New York

region is very thin, (Δr << r) and the area of the region can be approximated by 2pr(Δr) then the following equations are valid:

$$\frac{dP}{dx} = -\frac{2\tau_0}{(\Delta r)} \tag{1}$$

$$D_h = 2\left(\frac{area}{perimeter}\right) = \Delta r \tag{2}$$

$$\mathrm{Re} = \frac{D_h V \rho}{\mu} = \frac{Q\rho}{2\pi r \mu} \tag{3}$$

$$\tau_0 = c_f \frac{\rho V^2}{2} \tag{4}$$

$$V = \frac{Q}{2\pi r(\Delta r)} \tag{5}$$

From Kays and Crawford[Error! Bookmark not defined.] we have:

$$c_f = \frac{24}{\mathrm{Re}} \tag{6}$$

Substituting in for the pressure drop we have:

$$P_i - P_o = \frac{12QL\mu}{\pi r(\Delta r)^3} \tag{7}$$

The flow divider must generate a pressure drop at the desired full scale flow drives the proper flow through the sensor tube to generate a full scale output from the sensor. Since the full scale flow of the sensor is the same for all of the different full scale flows that may pass through the flow divider, the flow divider geometry must vary for the different full scale flows in order to generate the same pressured drop for all of them. From (17) it can be seen that if the width of the annular ring is varied slightly it can correct for very large changes in the full scale flow rate (Q).

Below is a graph showing how the thickness of the annular ring must be changed to create a passage that will properly divide the flow for various full scale flows. This graph is based on the 75 Pa pressure drop required to push full scale flow through a particular sensor that has 2 cm spacing between the inlet and outlet taps. The flow divider has an outside diameter of 0.95 cm.

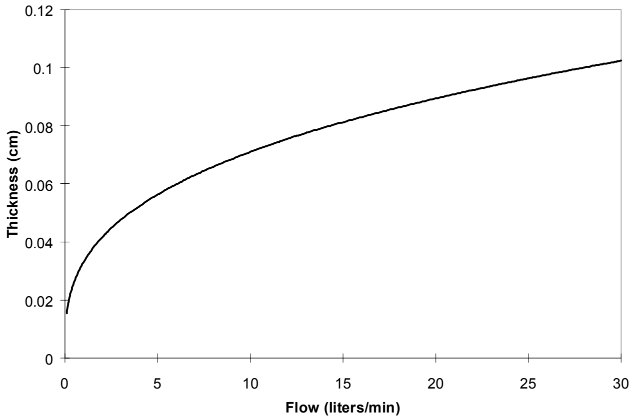

Figure 1 Thickness of the annular ring as a function of flow rate for a sensor with a 75 Pa drop and a 2 cm spacing.

Each flow divider must have a section of the annular region upstream of the upstream sensor tap to allow the flow to become fully developed before reaching the first tap. Using equation (7) we can determine the entry length for the annular passage as:

$$L_e = \frac{Q\rho(\Delta r)}{40\pi r \mu} \tag{8}$$

Below is a graph that demonstrates the entry length that would be required to design a flow divider for various full scale flows. The parameters on the sensor that the flow divider must match are the same as the ones on the previous graph.

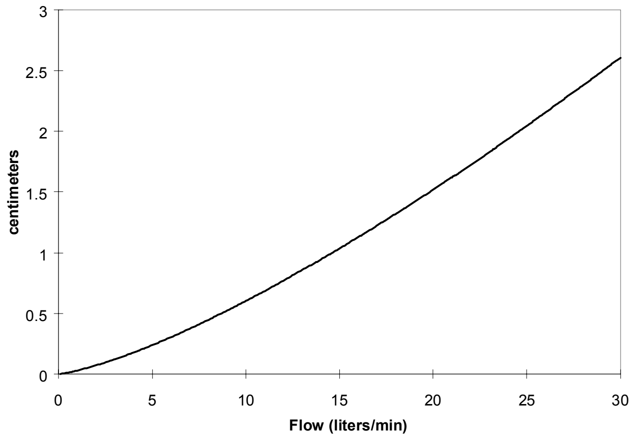

Figure 2 Entrance length as a function of flow rate for an annular ring of the size specified in figure 3.

CONCLUSION

By using these equations we can design a flow divider that has non-linear effects that are less than 1% of the total pressure drop at full scale flow for full scale flows up to 20 liters/min in a 2 cm diameter bore.

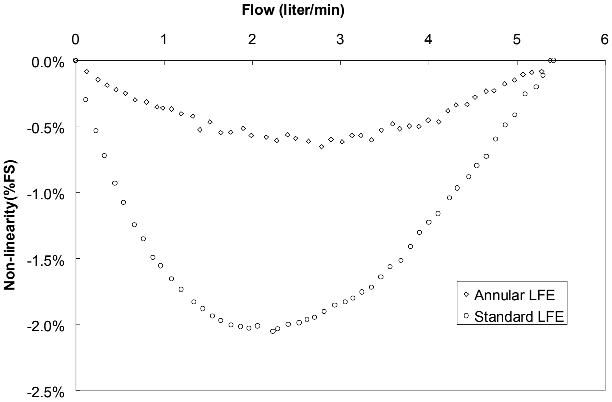

Figure 1 A comparison of the terminal based linearity between a multi-tube shunt with external sensor taps and an annular shunt with internal sensor taps.

If flows higher than this are desired it can easily be seen that multiple annular passageways with the same flow ring thickness can be built inside of each other to create parallel passageways. These passageways will all have the same volumetric flow versus pressure drop shape as the first channel. They will differ only in the magnitude of the flow that passes through each passageway for the a given pressure drop. The magnitude of the flow is proportional to the ratio between the flow areas of the channels. Flows up to 50 liters/min can be achieved this way in the same bore size as the previous shunt. However, the flow must be fully developed in the main bore before the face of the laminar flow element is reached, otherwise the flow will not be evenly divided among the flow channels.

Still higher flows can be measured by folding very thin metal foil into a cylinder containing a myriad of capillary flow channels that have approximately the same hydraulic diameter as the outer annular passageway. Laminar flow elements of this type can be used to measure flows of 1000 liters/minute with a linear pressure drop in a 4.4 cm bore.

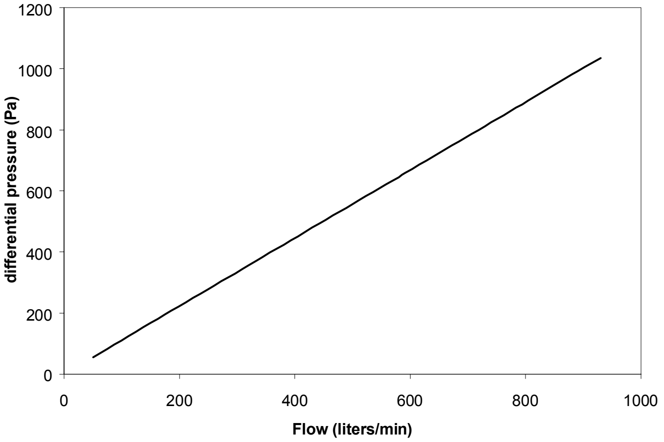

Figure 2 Pressure drop of laminar flow element designed for a sensor with 1000 Pa drop at full scale and a 2 cm tap spacing.

ACKNOWLEDGMENTS

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[1] Bernoulli, Daniel., Bernoulli, Johanne., 1968 “Hydrodynamics”, Dover Publications, New York

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[1] Kays, W. M., Crawford M. E., 1993, “Convective Heat and Mass Transfer”, 3rd edition, McGraw-Hill, New York